X-621-64-294

NASA TMX-55139

N65-18261
(ACCESSION NUMBER)
17
(PAGES)
TMX-55139
(NASA CR OR TMX OR AD NUMBER)
1
(THRU)
(CODE)
31
(CATEGORY)

FACILITY FORM 602

THE ROTATION OF SYNCOM III DURING LAUNCH

GPO PRICE $

OTS PRICE(S) $

Hard copy (HC) $1.00

Microfiche (MF) $0.50

OCTOBER 1964

GODDARD SPACE FLIGHT CENTER
GREENBELT, MD.

X-621-64-294

THE ROTATION OF SYNCOM III DURING LAUNCH

by

David L. Mott
Physical Science Laboratory
New Mexico State University

October 1964

Goddard Space Flight Center
Greenbelt, Maryland

THE ROTATION OF SYNCOM III
DURING LAUNCH

SUMMARY

18261

An analysis is made of telemetry data from Syncom III covering the launch ~~period from~~ spinup to third-stage/spacecraft separation. The early spin history of the satellite is given, and its torque-free motion before and after separation is described. The observed torque-free motion is shown to be consistent with theory. From the data, values of the ratio of roll-to-pitch moments of inertia are calculated both for the burned-out Delta third stage with payload and for the separated spacecraft.

Author

CONTENTS

ILLUSTRATIONS

INTRODUCTION

The Syncom III satellite was launched from Cape Kennedy on August 19, 1964. The launch vehicle was a thrust-augmented Delta, and liftoff was at 12 hours, 15 minutes, 1.8 seconds (GMT).

A schedule of events within the period covered by this report is given in Table I. The telemetry tape used in this analysis was recorded at Ascension Island, and the analysis is restricted to data consisting of the pulses from the spacecraft's ψ and ψ_2 sun sensors and the continuous signal from its accelerometer.

Table I
Partial Listing of Syncom III Launch Sequence

Hour (GMT)	MIN	SEC	EVENT
12	41	5	Spinup
		7	Second/third-stage separation
		12	Third-stage ignition
		34	Third-stage burnout
	42	41	Third-Stage/spacecraft separation

The sun-sensor pulses are used to determine the spin rate and orientation of the satellite with respect to the sun line. The slit of the ψ sun sensor is parallel to the satellite's axis of symmetry, and that of the ψ_2 sensor is canted 35 degrees. The distance between the ψ- and ψ_2-pulses, on a scale in which the distance between ψ-pulses is 360 degrees, is the angle ψ_2 (angle ψ_2 is positive if the ψ-pulse preceeds the ψ_2-pulse). The angle between the axis of symmetry and the line to the sun is ϕ (ϕ is less than 90 degrees when the sun is to the motor end of the satellite), and

$$\cot \phi = \sin \psi_2 \cot 35^\circ .$$

The accelerometer signal is constant in amplitude if the axis of rotation $\vec{\omega}$ is stationary in the body coordinate system (and if there is no linear acceleration(!)). If $\vec{\omega}$ is precessing in the body coordinate system, the amplitude changes as the distance from the accelerometer to $\vec{\omega}$ changes.

PARAMETERS OBTAINED FROM THE DATA

Figure 1 shows the ψ-pulse rate in rpm from spinup to shortly after third-stage burnout. The abcissa N is an index for the ψ-pulses, and the point at N = J gives the rpm indicated by the distance on the tape between pulses J and J - 1. Figure 1 includes the first 100 ψ-pulses on the tape. For the remainder of the tape, the rate is essentially constant at 166 rpm. There is no apparent perturbation of the rate at third-stage/spacecraft separation.

At spinup, the angle ϕ is nearly 90 degrees and the ψ- and ψ_2-pulses are not distinctly separated. However, it appears that ψ_2 preceeds ψ by some small amount less than one degree.

The angles ψ_2 and ϕ are constant until third-stage burnout, at which time an oscillation is suddenly initiated as shown in Figure 2. The maximum and minimum excursions of the angle ψ_2 are 8.5 degrees and -7.2 degrees which correspond to ϕ-angles of 78.1 and 100.1 degrees. This implies that the third stage with payload is coning* about an average ϕ of 89.1 degrees at a cone half-angle of 11.0 degrees. The frequency of coning is 18.1 rpm.

At the onset of the ϕ-oscillation, a distinct periodic amplitude modulation of the accelerometer signal commences. The modulation frequency is 150 rpm.

The coning remains constant in amplitude and frequency until 42 minutes, 41 seconds, at which time there is a sudden change as shown in Figure 3. The point of transition identifies the time of spacecraft separation from the third stage.

Since the roll moment of inertia of the spacecraft exceeds its pitch moment of inertia, the coning frequency of the separated spacecraft must exceed the rotation frequency (see "THEORY"); thus the ψ-pulses are not an adequate sample for determining the coning frequency. From the pulses following separation it can be estimated that the maximum and minimum excursions of ψ_2 are 8.0 degrees and 6.25 degrees which yield a cone half-angle of 1.2 degrees about an average ϕ of 80.0 degrees. In "CALCULATIONS," the coning frequency is calculated to be 204 rpm. This value is used in Figure 3 and it is seen that the recorded ψ_2 values agree excellently with this pattern.†

Following separation, the modulation of the accelerometer signal has a frequency of 38.5 rpm. The percent of modulation is very small because the center of gravity is now close to the accelerometer station.

*In space coordinates with origin at the center of gravity, the body axis of symmetry moves so as to describe the surface of a cone.

†This pattern would be apparent if there were ψ and ψ_2 sensors in each quadrant.

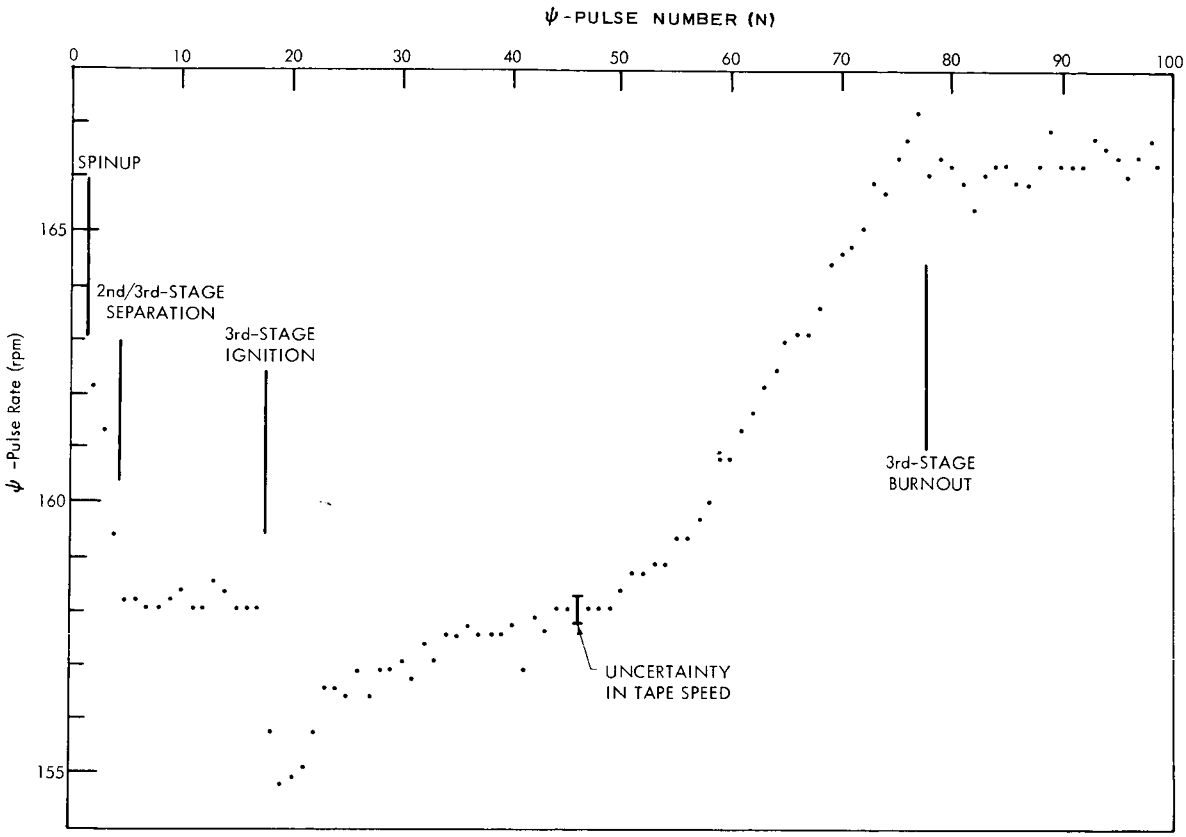


Figure 1-ψ-Pulse Rate from Spinup to Third-Stage Burnout

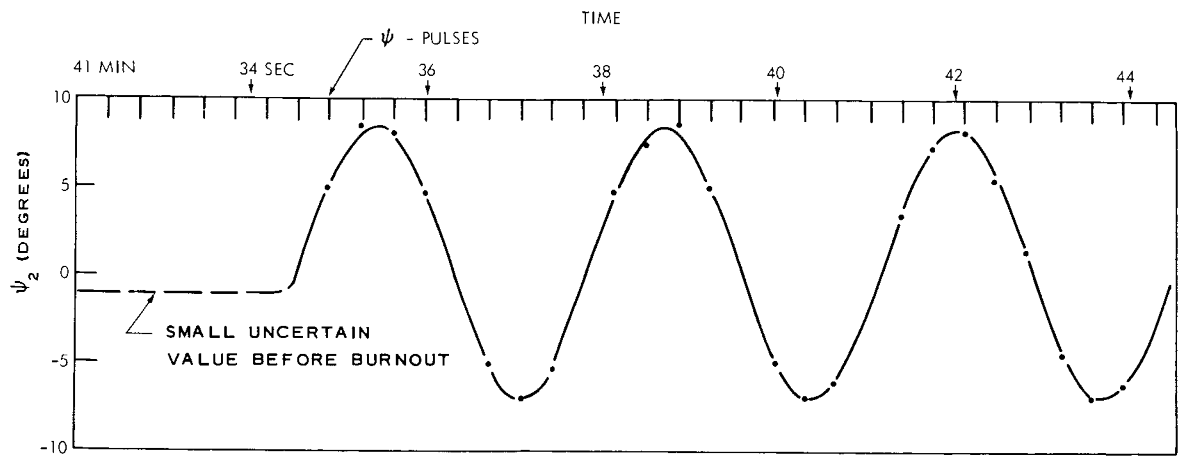


Figure 2-Angle ψ_2 Oscillation at Third-Stage Burnout

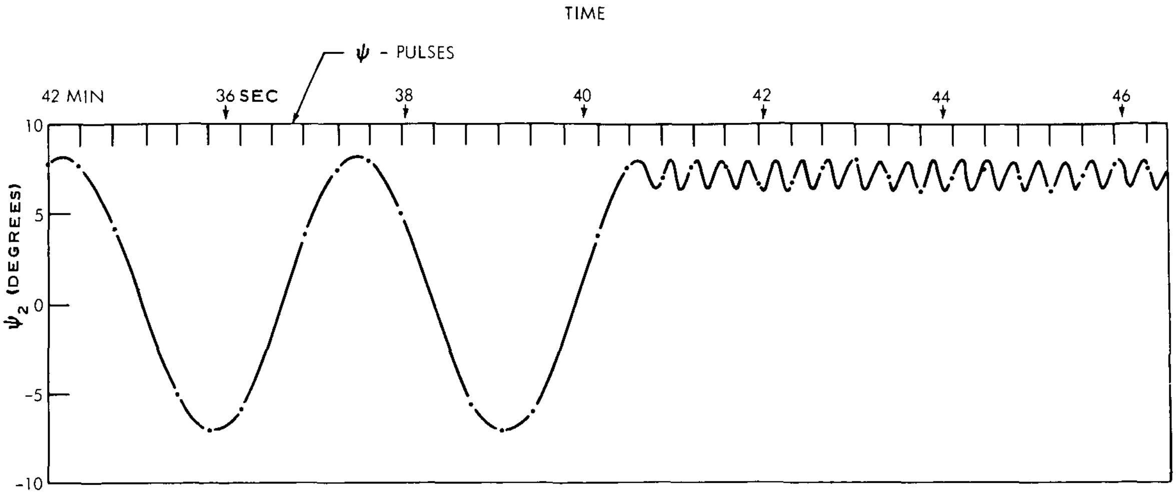


Figure 3-Angle ψ_2 Oscillation at Third-Stage/Spacecraft Separation

THEORY

Symbols

n   Precession frequency (the accelerometer signal modulation frequency, and the frequency at which the spacecraft's "nutation damper" is driven)

Ω Coning frequency

$\vec{\omega}$ Vector of the angular velocity

ω_ζ Component of $\vec{\omega}$ along the body axis of symmetry, ζ (the ψ-pulse rate)

$\vec{L}$ Vector of the angular momentum

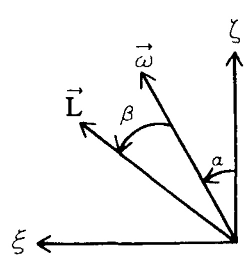
I_1 Pitch moment of inertia

I_3 Roll moment of inertia

m $m \equiv I_3/I_1$

α Angle between ζ and $\vec{\omega}$

β Angle between $\vec{\omega}$ and $\vec{L}$

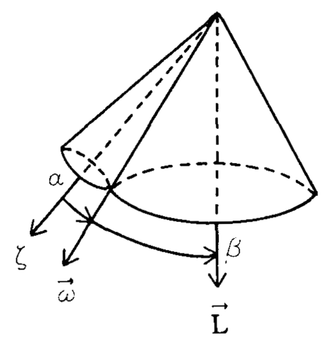


$$\omega_\zeta = \omega \cos \alpha, \ \omega_\xi = \omega \sin \alpha$$

$$\tan (\alpha + \beta) = \frac{L_\xi}{L_\zeta} = \frac{1}{m} \tan \alpha.$$

Case I, $m < 1$

The torque-free motion of a rotating body that is symmetrical about the ζ-axis is identical with that of the smaller cone shown in the accompanying figure which rolls without slipping on the larger, stationary cone. The vectors $\vec{\omega}$ and $\vec{L}$ are those of the rotating body; $\vec{\omega}$ is the instantaneous axis of rotation, and the angular momentum $\vec{L}$ is constant in torque-free motion. For the two cones, $\vec{\omega}$ is the line of contact. The time for this line of contact to sweep once over the surface of the fixed cone is $1/\Omega$, and Ω is the coning frequency (in revolutions per unit time). The time for the line of contact to sweep once over the surface of the rolling cone is $1/n$, and n is the precession frequency. Precession in this context means a change in the location of the axis of rotation in relation to the body axes. In Goldstein,* Ω has the meaning that n does here; thus



$$n = (1 - m)\ \omega_\zeta$$

according to his equation (5-40).

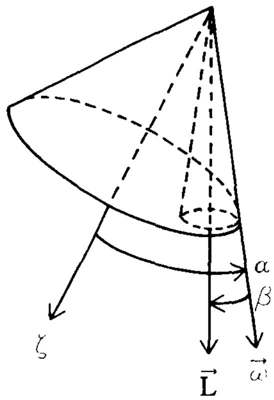
It is easy to verify that

$$\Omega/\omega = \sin \alpha / \sin (\alpha + \beta)$$

and

$$\Omega/n = \sin \alpha / \sin \beta.$$

*See References.



Case II, m > 1

The behavior of the body is identical with that of the larger cone in the accompanying figure which rolls without slipping about the smaller, stationary cone. The same results obtain for this case if β is considered negative; i.e.,

$$(\alpha + \beta) < \alpha.$$

CALCULATIONS

Coning Before Separation

From "PARAMETERS OBTAINED FROM THE DATA,"

$$\alpha + \beta = 11.0°$$

$$n = 150 \text{ rpm}$$

$$\Omega = 18.1 \text{ rpm}$$

$$\omega_\zeta = 166 \text{ rpm}$$

$$n = (1 - m)\,\omega_\zeta \rightarrow \boxed{m = 0.10}$$

$$\tan \alpha = m \tan (\alpha + \beta) = 0.10 \times 0.194 = 0.0194$$

$$\alpha = 1.1°, \ \beta = 11.0 - \alpha = 9.9°$$

Also,

$$n/\Omega = 8.29 = \sin \beta / \sin (11.0 - \beta) \rightarrow \beta = 9.8°$$

$$\alpha = 1.2°$$

These two evaluations of α and β verify that the observed values of n, Ω, and ω_δ are reasonably self-consistent.

In the Douglas Aircraft Company publication of "Detailed Test Objectives" for this launch (April 1964, Fig. 22), the following values are indicated for the burned-out third stage and spacecraft:

$$I_1 = 32 \text{ slug feet}^2$$

$$I_3 = 2.9 \text{ slug feet}^2$$

For these values, m = 0.09.

It is helpful to understand the physical picture of the coning. Prior to burn-out, $\vec{\omega}$ and $\vec{L}$ are on the axis of symmetry ζ. The situation is changed as some physical phenomenon shifts $\vec{\omega}$ by 1.2 degrees (angle α) within the body. With a new axis of rotation, there is a new – and now non-diagonal – inertia tensor $\mathbb{I}$. The shift of $\vec{L}$ to 11.0 degrees (angle $\alpha + \beta$) from the ζ-axis must be in accord with the equation $\vec{L} = \mathbb{I}\vec{\omega}$, and now the axis of symmetry begins to cone about $\vec{L}$ as explained in "Theory." The immediate effect of the physical phenomenon is not of necessity a sudden shift of the body orientation, but only an offset of the $\vec{\omega}$-vector by 1.2 degrees within the body. In other words, the cone about $\vec{L}$ described by the ζ-axis can very well be tangent to the pre-coning orientation of the ζ-axis.

Coning After Separation

From "PARAMETERS OBTAINED FROM THE DATA,"

$$\alpha + \beta = 1.2^\circ$$

$$n = (-)\ 38.5 \text{ rpm}$$

$$\omega_\zeta = 166 \text{ rpm}$$

$$n = (1 - m)\,\omega_\zeta \rightarrow \boxed{m = 1.23}$$

In the Hughes Aircraft Company publication of "Syncom C System Summary" (April 1964, Table 9.1), the values for the payload at separation are:

$$I_1 = 1.76 \text{ slug feet}^2$$

$$I_3 = 2.20 \text{ slug feet}^2$$

$$m = 1.25$$

Using the value $m = 1.23$,

$$\tan \alpha = m \tan(\alpha + \beta) = 1.23 \times 0.0209 = 0.0257$$

$$\alpha = 1.5°, \ \beta = 1.2 - \alpha = -0.3°$$

Note that the act of separation has little effect on α.

$$\omega = \omega_\zeta / \cos \alpha \sim \omega_\zeta$$

$$\Omega = \omega \frac{\sin \alpha}{\sin(\alpha + \beta)} \sim \omega \frac{\tan \alpha}{\tan(\alpha + \beta)} \sim m\omega_\zeta$$

$$\Omega = 1.23 \times 166 = 204 \text{ rpm.}$$

Table II summarizes these results, and the results from "CONING BEFORE SEPARATION."

Table II
Summary of Calculations

	ω_ζ	n	Ω	$\alpha+\beta$	α	β	m
Before Separation	166 rpm	150 rpm	18.1 rpm	11.0°	1.2°	9.8°	0.10
After Separation	166 rpm	-38.5	204	1.2	1.5	-0.3	1.23

Energy Considerations

The rotational kinetic energy of a body that is symmetrical about the ζ-axis, as a function of the Euler angles (ϕ, ψ, θ), is (Goldstein, eq. 5-43):

$$T = \frac{1}{2} I_1 (\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) + \frac{1}{2} I_3 (\dot{\psi} + \dot{\phi} \cos \theta)^2,$$

and

$$\omega_\zeta = \dot{\psi} + \dot{\phi} \cos \theta .$$

For our application, $\dot{\phi} \to \Omega$, $\theta \to (\alpha + \beta)$, and $\dot{\theta} = 0$.

Thus

$$T = \frac{1}{2} I_1 \Omega^2 \sin^2(\alpha + \beta) + \frac{1}{2} I_3 \omega_\zeta^2$$

There are at least three possible causes for the coning which commences at third-stage burnout. These are:

1. Deformation of the hot third stage
2. Fragments thrown from the third stage
3. Thrust misalignment of the dying flame

Let the rotational energy <u>just</u> before burnout be T_i , and after burnout be T_f : $\Delta T \equiv T_f - T_i$. If 1. were the cause, ΔT would be nearly zero; if 2. were the cause, ΔT would be negative; and if 3. were the cause, ΔT would be positive.

The critical factor in evaluating ΔT is ω_ζ , the ψ-pulse rate. From Figure 1, $\omega_{\zeta f}$ (after burnout) is nearly 166 rpm, or 17.4 radians per second.

$$I_1 = 32 \text{ slug ft}^2$$

$$I_3 = 2.9 \text{ slug ft}^2$$

$$(\alpha + \beta)_f = 11.0^\circ, \ \sin 11.0^\circ = 0.191$$

$$\Omega_f = 18.1 \text{ rpm} = 1.89 \text{ radians/sec.}$$

Thus
$$T_f = \frac{1}{2} I_1 \Omega_f^2 \sin^2(\alpha + \beta)_f + \frac{1}{2} I_3 \omega_{\zeta f}^2$$

$$T_f = (2.08 + 439) \text{ slug ft}^2/\text{sec}^2.$$

Figure 1 suggests that the value of ω_ζ, as it increases during third-stage burning, reaches a peak of about 167 rpm; i.e., $\Delta\omega_\zeta = \omega_{\zeta f} - \omega_{\zeta i} \sim 166 - 167 = -1$ rpm = -0.105 radians/sec.

Now $(\alpha + \beta)_i = \Omega_i = 0$, so that

$$\Delta T = \frac{1}{2} I_1 \Omega_f^2 \sin^2 (\alpha + \beta)_f + \frac{1}{2} I_3 (\omega_{\zeta f}^2 - \omega_{\zeta i}^2)$$

$$\Delta T \sim 2.1 \text{ slug ft}^2/\text{sec}^2 + I_3 \omega_{\zeta f} \Delta\omega_\zeta .$$

For $\Delta\omega_\zeta$ = -.105 radians/sec, $I_3\omega_{\zeta f}\Delta\omega_\zeta$ = -5.3 slug ft^2/sec^2, and ΔT = 2.1 - 5.3 = -3.2 slug ft^2/sec^2.

Assigning the value of -1 rpm to $\Delta\omega_\zeta$ really is based on the point in Figure 1 at N = 77. It is not impossible that this point is high by about the same amount that the point at N = 82 is obviously low. If this were true, $\Delta\omega_\zeta$ would be zero and ΔT = +2.1 slug ft^2/sec^2 would be obtained.

From these considerations, it would not be reasonable to say with certainty that the rotational kinetic energy either increased or decreased at the commencement of coning.

Inertia Considerations

Although the preceeding considerations of energy fail to point out the cause of the coning at third-stage burnout, assume for the sake of argument that it was due to the loss of an increment of mass δm.* An approximate calculation can be made to determine the order of magnitude of this mass.

*Or to any equivalent unbalance.

Prior to coning, the third stage with payload is spinning on the ζ-axis, and ζ is a principal axis. At the instant δm is lost, $\vec{\omega}$ is still along ζ, but ζ is no longer a principal axis. Let ζ' be the new principal axis, and the angle between ζ and ζ' must be 1.2 degrees (angle α).

The mass increment $-\delta m$ is assumed to be so small that I_1 and I_3 do not need to be re-evaluated. Let δm leave from the body coordinates (ξ, ζ). Then at the instant δm is lost, the inertia tensor is approximately

$$\mathbb{I} = \begin{bmatrix} I_1 & 0 & \epsilon \\ 0 & I_1 & 0 \\ \epsilon & 0 & I_3 \end{bmatrix}$$

where $\epsilon = \delta m \xi \zeta$. Solving for the eigenvectors of the operator $\mathbb{I}$, and using the inequality $\epsilon^2 << (I_1 - I_3)^2$, it is found that

$$\tan \alpha \sim \frac{\epsilon}{I_1 - I_3} .$$

Thus $\epsilon \sim \tan 1.2° \times (32 - 2.9) = 0.61$ slug ft^2;

$$\delta m \sim \frac{20}{\xi \zeta} \text{ lbs} \quad (\xi \text{ and } \zeta \text{ in feet}).$$

The maximum value for ζ is about 6 feet (from the cg of the third stage with payload to the aft end) and for ξ, 3/4 feet (radius of tank). Thus, a realistic value for a minimum δm would be on the order of 5 pounds

CONCLUSIONS

The analysis of the torque-free motion before and after separation is an interesting exercise in classical mechanics, and the results are reasonably self-consistent. Although text books deal thoroughly with the subject, experimental data are not readily obtained in earth-bound laboratories.

The data telemetered from the satellite does not in itself seem sufficient to explain the shift in $\vec{\omega}$ at third-stage burnout, and additional information is needed to resolve the question.

REFERENCES

Bracewell, R.N. and Garriott, O.K., Nature 182, 760 (1958).

Goldstein, H., Classical Mechanics, Addison Weekly Publishing Co., Reading, Mass., 1950, Chapter 5.